CENTER FOR BEAM PHYSICS SEMINAR

"Why the Symplectic Group Is Good for Classical Mechanics: A Pedagogical and Evangelical Talk"

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Friday February 15, 2002, 10:30 AM
Albert Ghiorso Conference Room (71-264), LBNL
Refreshments served at 10:20 AM

Abstract: Historically there are several mathematical groups that have been studied in detail because of their relevance to our understanding of the physical world. A detailed knowledge of the 3-dimensional rotation group is of great use in many areas including condensed matter physics, chemistry, atomic physics, nuclear physics, and elementary particle physics. Knowledge of the rotation-translation group leads to a classification of crystals and quasicrystals. Knowledge of the unitary group is helpful for quantum mechanics, and essential to an understanding of quantum information and quantum computing. Knowledge of the Lorentz group leads to the construction of spinors, 4-vectors, general tensors, and classical fields. Knowledge of the Poincaré group (the Lorentz group plus translations in space and time) leads to a classification of elementary particles and the construction of quantum fields. Knowledge of various other groups facilitates many-body theory calculations. Finally, there are the various "internal" and/or gauge symmetry groups that play an important role in our current understanding of elementary particles and the fundamental forces. The symplectic group is the underlying group in Classical Mechanics for Hamiltonian systems. Yet, in contrast to the groups just mentioned, almost nothing is commonly known or readily available about the symplectic group. For example, many are familiar with some aspects of the rotation group including spin (irreducible representations and how they are labeled) and how spins couple and combine (the Clebsch-Gordan series and coefficients for the rotation group). Yet few have heard or read about representations of the symplectic group, knowledge of its Clebsch-Gordan series is not widespread, and little is known in detail about its Clebsch-Gordan coefficients. The purpose of this talk is to describe some aspects of the finite-dimensional representations of the first few symplectic groups with the hope that this knowledge, like that for the well-studied groups, will also ultimately prove useful. Indeed, as a first consequence of this effort, we will find a symplectic classification of all polynomial vector fields (including those for systems with dissipation), and a deeper understanding of beam moment and emittance invariants.

Biographical Sketch: Alex Dragt is Professor of Physics at the University of Maryland where he heads a Dynamical Systems and Accelerator Theory Research Group, and is currently spending a 6-month sabbatical leave at LBNL. He comes from a particle theory background (thesis work at Berkeley) and has interests in nonlinear dynamics, group theory, and more recently in quantum computing. Related Web sites are www.physics.umd.edu/dsat and www.physics.umd.edu/sqc